ARJUNA

JEE

LECTURE NUMBER - 7

Basic Math

MSM SIR

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"New cheez"

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QUESTIONS PRACTICE

Q. Solving the inequality

$(x^2 - 3x + 2)(x^3 - 3x^2)(4 - x^2) \geq 0$ **(HW)**

$\times(-1)$

$(-)(x^2-3x+2)(x^3-3x^2)(4-x^2) \leq -0$

$(x^2-3x+2)(x^3-3x^2)(x^2-4) \leq 0$

$(x-1)(x-2)\,x^2(x-3)(x-2)(x+2) \leq 0$

$x^2(x-2)^2(x-3)(x+2)(x-1) \leq 0$

NoNe, CB

$x = 0$

$x = 2$

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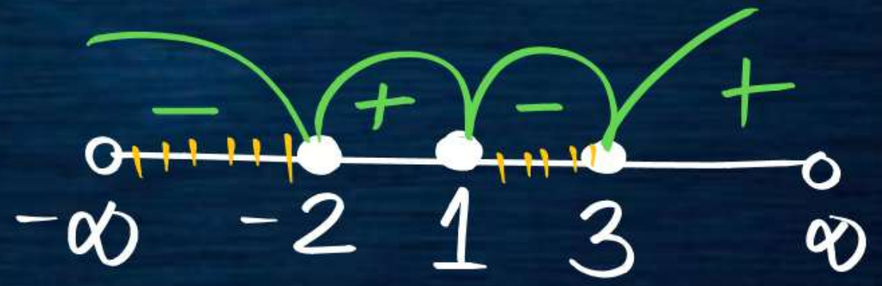
$x = 0, 2$

$$(x-3)(x-1)(x+2) \leq 0$$



$$x \in (-\infty, -2] \cup [1, 3] \cup \{0\}$$ Ans

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MORE No-Ne Expressions

APo ⇒ Always positive → Positive ✓
→ zero ✗
→ Negative ✗

example ⇒ Quadratic
" ax^2+bx+c "
and $\begin{cases} a>0 \\ D<0 \end{cases}$
⇒ $ax^2+bx+c>0$ → APo, CB
(cannot be broken into linear factors)

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Q. Solving the inequality

$$\frac{x}{x-2} > \frac{1}{x+3}$$

?

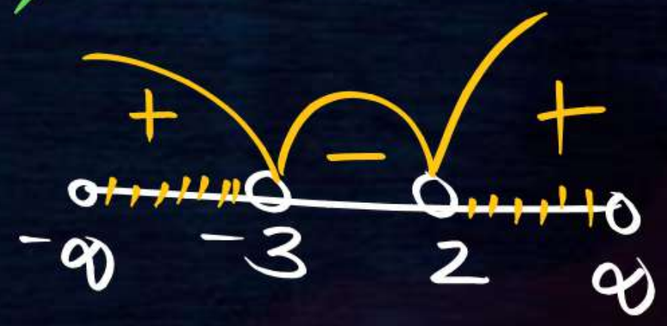
$$\frac{x}{x-2} - \frac{1}{x+3} > 0$$

$$\frac{x(x+3)-(x-2)}{(x-2)(x+3)} > 0$$

$$\frac{(x^2+2x+2)}{(x-2)(x+3)} > 0$$ APo, CB

$$\begin{cases} a=1 \\ D=4-8 \\ D<0 \end{cases}$$

$$\frac{1}{(x-2)(x+3)} > 0$$



$x \in (-\infty, -3) \cup (2, \infty)$

Ans

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Q. Solving the inequality

$$\frac{3}{x-1}-\frac{2}{x+1}\geq 1$$

"Never Cross Multiply"

?

$$\frac{3}{x-1}-\frac{2}{x+1}-1\geq 0$$

$$\frac{3(x+1)-2(x-1)-(x+1)(x-1)}{(x-1)(x+1)}\geq 0$$

$$\frac{x+5-x^2+1}{(x-1)(x+1)}\geq 0$$

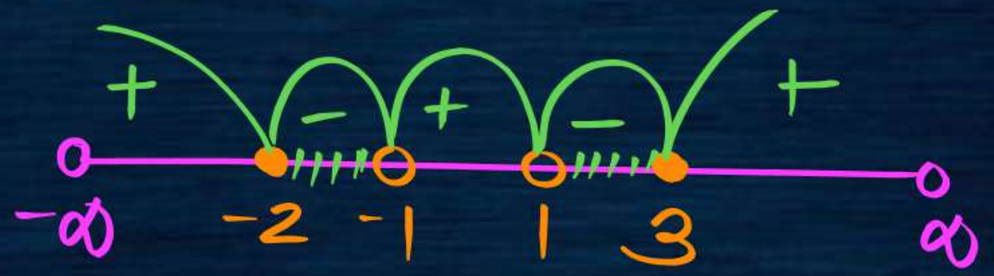
$$\frac{(-x^2+x+6)}{(x-1)(x+1)}\geq 0$$

$\times(-1)$

$$-\left(\frac{-x^2+x+6}{(x-1)(x+1)}\right)\leq 0$$

$$\frac{(x^2-x-6)}{(x-1)(x+1)}\leq 0$$

$$\frac{(x-3)(x+2)}{(x-1)(x+1)} \leq 0$$



$x \in [-2,-1) \cup (1,3]$ Ans

Q. Solving the inequality

$$\frac{(x^2-3x+2)(x^2+2x+2)}{(x-3)(x-1-2x^2)} \geq 0$$

"Challenge" ?

$\times(-1)$

$$\frac{(x^2-3x+2)(x^2+2x+2)}{(-1)(x-3)(-2x^2+x-1)} \leq 0$$

$a=1, D<0$ → AP, CB

$$\frac{(x^2-3x+2)(x^2+2x+2)}{(x-3)(2x^2-x+1)} \leq 0$$

$a=2$

$D=1-8=-7;\ D<0$

AP, CB

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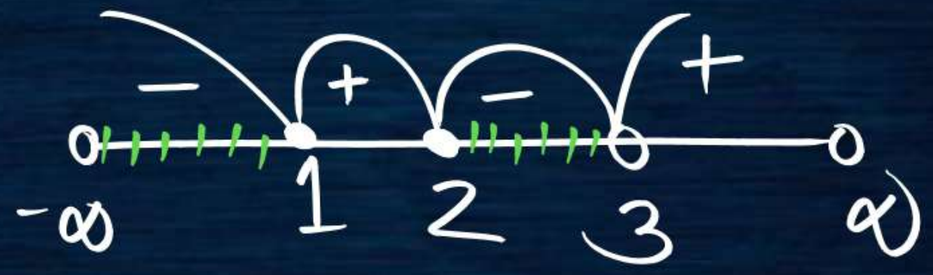
$$\frac{x^2-3x+2}{x-3} \leq 0$$

$$\frac{(x-1)(x-2)}{x-3} \leq 0$$



$x \in (-\infty, 1] \cup [2, 3)$ Ans

Q.

means integral values

Find integral 'x' satisfying:

$x^4 - 3x^3 - x + 3 < 0$

$x^3(x-3) - 1(x-3) < 0$

$(x-3)(x^3-1^3) < 0$

$(x-3)(x-1)(x^2+1+x) < 0$

$a=1,\ D=-3$

$a>0,\ D<0$

AP_0, CB

$(x-3)(x-1) < 0$

$x \in (1,3)$

Solve for 'x'

'interval of x'

'Range of x' ✓

'exhaustive values of x' ✓

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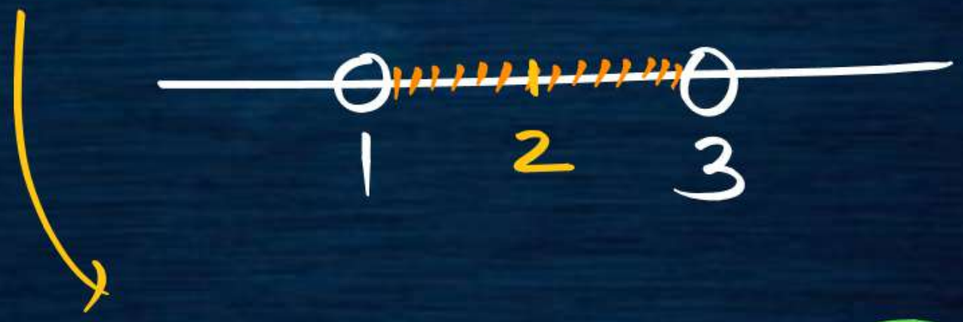
$x \in (1,3)$



Ans = integer ($x \Rightarrow$ "2")

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Q. Solving the inequality

$x^4 + 6x^3 + 6x^2 + 6x + 5 < 0$ (CHALLENGE - α guru)

Done

?

GEOMETRICAL INTERPRETATION OF MODULUS

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$|x-a|$ represents the distance between x and a on the number line.

ex → $|x-2|$ = Distance b/w 'x' and '2' on the number line.

- $|x+1| = |x-(-1)| = |x-a|$ = Distance b/w 'x' and '−1' on the number line.

- $|1-x|$ = Distance b/w '1' and 'x' on the number line.

- $|x| = |x-0|$ = Distance b/w 'x' and '0' on the number line.

$$|x-a| = |a-x|$$ ★

$|\underbrace{x-a}_{\alpha}| = |\underbrace{-(x-a)}_{-\alpha}|$

$$|\alpha| = |-\alpha|$$ ★

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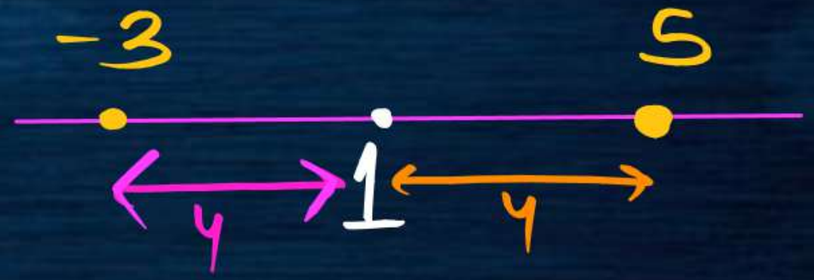
Q. $|x-1| = 4$, Solve for x.



$x \in \{-3, 5\}$

Ans

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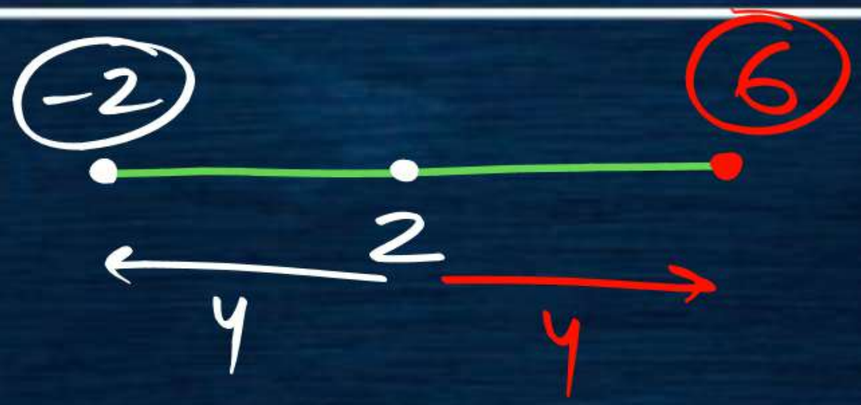
Q. $|2 - x| = 4$, Solve for x.



$x \in \{-2, 6\}$ Ans

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lengthy method

(Cases Banakar) already you Know how to do!

(DOUBLE MODULUS PROBLEMS)

Now Trick Develop Karenge

TYPE : $|x-a|+|x-b|=c$

(*SOCH-VICHAR WALLAH PART*)

Mei nahi sochunga
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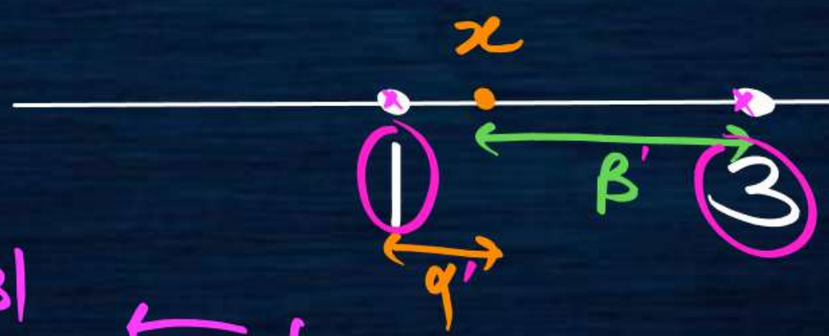
Q. $|x-1|+|x-3|=4$, *Solve for x.*

✓ Method 1 → already Done (Cases Banakar)

✓ Method 2 → "Bhokaadi Method"



$|x-1|+|x-3|$
$= \alpha' + \beta'$
$= |1-3|$
$= 2$

$\leftarrow \begin{cases} |x-1| = \alpha' \\ |x-3| = \beta' \end{cases}$

x cannot lie b/w '1' and '3'

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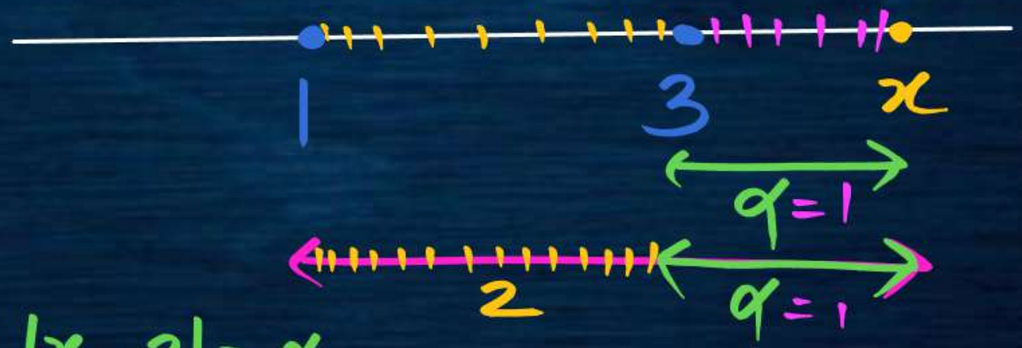

assume 'x' lies to the Right of '3'.

$$\begin{cases} |x-3| = q \\ |x-1| = q+2 \end{cases}$$

$|x-3| + |x-1| = 4$

$q + q + 2 = 4$

$2q = 2$

$\boxed{q = 1}$

$x = 4$

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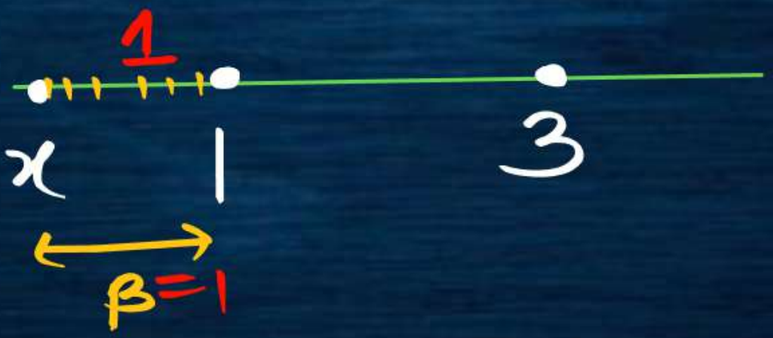
'x' lies to the left of '1'



$|x-1| = \beta$

$|x-3| = \beta + 2$

$|x-1| + |x-3| = 4$

$\beta + \beta + 2 = 4$

$2\beta = 4 - 2$

$\boxed{\beta = 1}$

$x = 0$

$x \in [0, 4]$ Ans

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Q-3 ⇒ **JBK method** → 'x' must not lie b/w 'a' and 'b'

For Format like →

$a < b$

$$|x-a| + |x-b| = c$$

① First Find → $\frac{c-|a-b|}{2} = 'd'$ ✓

② Ans → $x \in \{b+d;\ a-d\}$

Q → $|x-1|+|x-3|=4$, Find exhaustive values of 'x'.

$a=1 ; b=3 ; c=4$

$$d=\frac{4-|1-3|}{2}=\frac{4-2}{2}=(1)=d$$

Ans → $x \in \{4,0\}$

Q. $|x-2|+|x-3|=5, Solve\ for\ x.$

$a=2$

$b=3$

$c=5$

$\Rightarrow d = \dfrac{5-|2-3|}{2} = \dfrac{5-1}{2} = 2$

$x \in \{5, 0\}$ Ans

Q→ Solve for x →

$$|x-7|+|x-10|=6$$

$a=7;\ b=10;\ c=6$

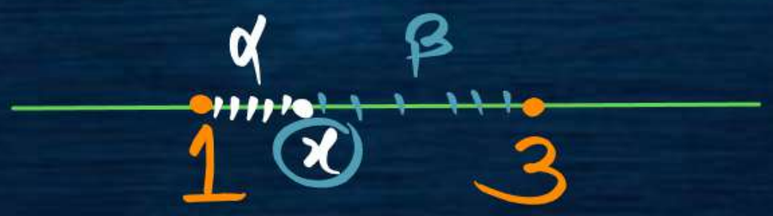
$$d=\frac{6-3}{2}=(1.5)$$

$$x\in\{5.5,\ 11.5\}$$ Ans

Q. $|x-1|+|x-3|=2$; Solve for x

?

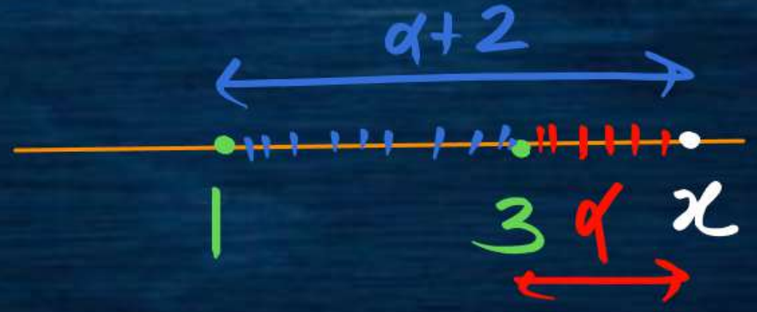
'x' lies b/w 1 and 3 ✓



$\alpha+\beta=2$

$|x-1|+|x-3|=2$

$x \in [1,3]$ Ans ✓



Checking whether 'x' lies to the Right of '3'

No

$|x-1| = (\alpha+2) > 2$

$(|x-3| = \alpha)$

$|x-1| + |x-3| = 2\alpha+2 > 2$

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checking whether 'x' lies to the left of "1"

"No"

$|x-3| > 2$

$|x-1|$

$|x-1| + |x-3| > 2$

BABA KA GYAAN:

→ modified JBK method..

If $|x-a|+|x-b|=c$, then
we can say that $|x-a|+|x-b|=c$ has at-least one solution if $|a-b| \leq c$.

↳ "next page"

Double Modulus Problem →

$$|x-a|+|x-b|=c$$

Step 1 ⇒ Find $|a-b|$

Step 2 ⇒ If $c>|a-b|$ → $d=\frac{c-|a-b|}{2}$

$x \in \{a-d, b+d\}$

$a<b$

If $c=|a-b|$ → $x \in [a,b]$

If $c<|a-b|$ → $x \in \phi$

PRODIGY CLASSES
PRODIGY CLASSES
Bhaiya aap kitna drama karte hain

Q. $|x-7|+|x-10|=3$, Solve for 'x'.

✓ $a=7$
✓ $b=10$
✓ $c=3$

$|a-b|=3=c$

$x \in [7,10]$ Ans

?

Q. $|x-7|+|x-10|=6$, Solve for 'x'.

$a=7$

$b=10$

$c=6$

$|a-b|=3$

$c>|a-b|$

$d=\frac{6-3}{2}=1.5$

$x\in\{5.5, 11.5\}$ Ans

Q. $|x-2|+|x-6|=2$, Find exhaustive values of 'x'.

$a=2$

$b=6$

$c=2$

$|a-b|=4$

$c=2$

$c<|a-b|$

$x\in\phi$ Ans

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Q → Solve for x →

(i) $|x-1|+|x-19| = 6$

$a=1,\ b=19$

$|a-b|=18$

$c<|a-b|$

↳ No soln

$x \in \phi$ Ans

(ii) $|x-1|+|x-19|=18$ → $a=1;\ b=19 \Rightarrow |a-b|=18;\ c=18$

$c=|a-b|$

$x \in [1,19]$ Ans

(iii) $|x-1|+|x-19|=20$

↳ $a=1,\ b=19;\ |a-b|=18$

$c=20$

$c>|a-b|$

$d=\frac{20-18}{2}=1$

Ans → $x \in [0,20]$ Ans

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Note →

TRICK is only for Formats like $|x-a|+|x-b|=c$;

In case of any other Format, always use '**Method 1**"

Aukaat yaad karwau?

Q. $|x-2|+|x-4| \geq 6$, Solve for x...

(OP Done)

(HW)

?

DPP DOUBTS (POPULAR)

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BABA KA GYAAN

$$x^2 = |x|^2$$

$$(\text{mmm})^2 = |\text{mmm}|^2$$

ex→ $\text{mmm} = x-3$

$$(x-3)^2 = |x-3|^2$$

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DPP4

9. $(x-3)^2 + |x-3| - 11 = 0$ the sum of solutions of equation is

(1) 5 (2) 6

(3) 7 (4) 8

$(x-3)^2 = |x-3|^2$

$|x-3|^2 + |x-3| - 11 = 0$

$|x-3| = t$

$t^2 + t - 11 = 0$

$t = \frac{-1 \pm \sqrt{1+44}}{2}$

$t = \frac{-1+\sqrt{45}}{2} = |x-3|$

$t = \left(\frac{-1-\sqrt{45}}{2}\right) = |x-3|$

Discarded

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$$|x-3| = \frac{\sqrt{45}-1}{2}$$

$$(x-3) = \pm\left(\frac{\sqrt{45}-1}{2}\right)$$

$\oplus$ $\qquad$ $(-)$

$$x-3 = \frac{\sqrt{45}-1}{2} \qquad x-3 = -\left(\frac{\sqrt{45}-1}{2}\right)$$

$$\boxed{x = 3+\frac{\sqrt{45}-1}{2}} \checkmark \qquad \boxed{x = 3-\left(\frac{\sqrt{45}-1}{2}\right)} \checkmark$$

add = 6.

DPP4

8. Solve: $x^2 - 7|x| - 8 = 0$ then value(s) of x satisfying the equation are

(1) ± 1 (2) ± 8

(3) ± 9 (4) ± 10

$x^2 = |x|^2$

$|x|^2 - 7|x| - 8 = 0$

$|x| = t$

$t^2 - 7t - 8 = 0$

$t = 8, -1$

$|x| = 8$; ~~$|x| = -1$~~

$x = \pm 8$

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DPP5 (CHALLENGE WALLAH SAWAAL)

MSM

4. Suppose $abc > 0$ and $a + b + c < 0$ and $\frac{|a|}{a} + \frac{|b|}{b} + \frac{|c|}{c} = x$ then value of $x^3 - 6x^2 + 11x - 6$ equals

(1) 0 (2) –24

(3) 12 (4) 10

$-1-6-11-6$

Put

$abc > 0$

~~all 3 positive~~

OR

any 2 Negative and 1 positive

$a+b+c<0$

$a<0, b<0, c>0$

$|a| = -a$

$|b| = -b$

$|c| = c$

$(a\ b\ c) > 0$

a	b	c
P	P	P
~~N~~	~~P~~	~~P~~
~~P~~	~~P~~	~~N~~
~~P~~	~~N~~	~~P~~
N	N	P
N	P	N
P	N	N
~~N~~	~~N~~	~~N~~

$x = \frac{-a}{a} + \frac{-b}{b} + \frac{c}{c}$

$= -1 - 1 + 1$

$x = -1$

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DPP 4 → complete

HOMEWORK SLIDE

JBK...JBK...JBK

- TO *FUTURE IIT-IANS*